MATHCOUNTS®

2017
■ Mock National Competition ■
Sprint Round
Problems 1-30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature ______________________________ Date ____________________

Printed Name ____

State____

DO NOT BEGIN UNTIL YOU HAVE SET YOUR TIMER TO FORTY MINUTES.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books, or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

1.________________ Let a⌘b=a+b+(a-1)⌘(b-1), and 1⌘2=3, find 4⌘5.

2.________________ Evaluate $2017 - \frac{2000\cdot 2034}{2017}$. Express your answer as a common fraction.

3.________________ The statistics for a test given to 30 students is shown below.

Score	# of students
<70%	5
70%-79%	6
80%-89%	6
90%-96%	9
97%-100%	4

Find the probability that a randomly selected pair of students Both have scores in the 97%-100% range. Express your answer as a common fraction.

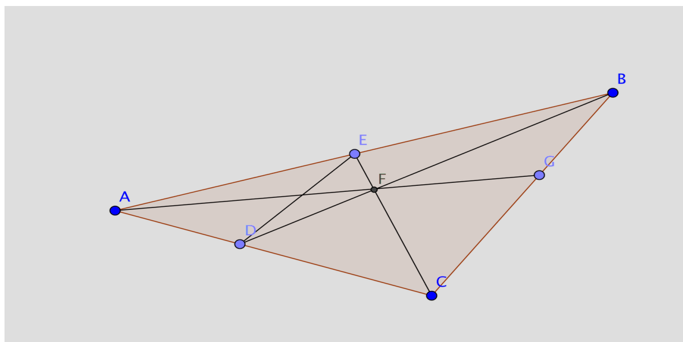
4.________________ Jack is driving to work for his house, a distance of 10 miles. On his way to work, he averages 50 mph. On the return trip, he encounters heavy traffic, taking him 12 minutes longer on the return trip. Assuming he drives at the same speed on the return trip, what is was his average speed throughout the whole trip in miles per hour? Express your answer as a common fraction.

5.________________ Let S be the set of all positive integers less than or equal to 101. Al is randomly assigned a subset A of S, and Betty is assigned the subset B such that B is all the elements in S that are not in A. (For example, if $A = \{1, 2, 3, \ldots, 49\}$, then $B = \{50, 51, 52, \ldots\ 101\}$.) What is the probability that the sum of the elements in each of their sets are equal?

6.________________ If the product of all factors of 9000 can be written in the form $2^a * 3^b * 5^c$ where a, b, and c are positive integers, find the sum $a + b + c$.

7.________________ Three dice, one red, one blue and one green die, are all tossed. Find the probability that the sum obtained was 17. Express your answer as a common fraction.

8.________________ In ΔABC, points D and E are chosen on segments $\overline{AC}$ and $\overline{AB}$ such that $\overline{DE}$ is parallel to $\overline{BC}$, as shown below. The ratio $\overline{AD} : \overline{CD}$ is $\frac{3}{4}$. If AF is extended to BC, and intersects it at G, what is the ratio $\overline{CG} : \overline{GB}$? Express your answer as a ratio a:b, where $(a, b) = 1$.



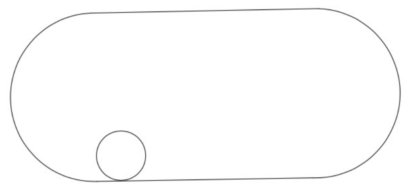
9.________________ All the dates in the year 2017 are listed in one long string:
1 2 3 4 5 6 7 8 9 10 11 … 31 1 2 3 4 5 6 7 8 …
How many times does 3 appear in the string?

10.________________ What is the size of the largest subset, S , of {1, 2, 3 ,4 … 51, 52} such that no pair of distinct elements of S has a sum divisible by 7?

11. ________________ A women with a basket of eggs finds that if she removes either 2, 3, 4, 5, or 6 at a time from the basket, there is always one egg left over. If she removes 7 eggs at a time from the basket, there are no eggs left over. If the basket holds between 500-1000 eggs, how many eggs does the basket hold?

12. ________________ A circle is inscribed inside a triangle with sides of length 10, 17, and 21. The same triangle is then inscribed inside another circle. What is the ratio of the radius of the inscribed circle to the ratio of the circumscribed circle? Express your answer as a common fraction.

13._________________ In the diagram shown below, a circle is being rolled clockwise along the inside of the track with parallel sides of length 4 cm, and congruent semicircles.



The perimeter of the track is $4\pi + 8$ cm, and the radius of the Circle is 1 cm, find the distance the center of the circle travels after one revolution around the track? Express your answer in terms of π.

14. _________________ Find value of a that satisfies $\dfrac{18a+1}{27a+1} = \dfrac{20a+1}{30a+3}$. Express your answer as a common fraction.

15. _________________ If the sum of the digits of 13^{99} can be expressed as $9m + n$, where $0 \le n < 9$, find n.

16. _________________ Alice and Betty run a 20 mile race. Alice runs at a constant speed of 6 mph, while Betty runs at 4 mph. Given that they each randomly start to run at a time between 9:00 and 11:00, what is the Probability that Betty finishes the 20 mile race before Alice? Express your answer as a common fraction.

17. _________________ How many positive 6-digit integers, whose digits are positive integers less than 7, are divisible by 4?

18. ________________ How many 3-digit positive integers are there such that their digits are in a non-increasing order from left-right? For example 321,331,333 are included, but not 334.

19. ________________ A tetrahedron ABCD has three faces with right triangles whose right triangles share a same vertex A. If $\overline{AB}$ =2, $\overline{AD}$=4, and $\overline{AC}$=3, find the height from vertex A to triangle BCD. Express your answer as a common fraction in simplest radical form.

20. ________________ Al rolls a fair blue dice, then a fair red dice. Let b and r be the two numbers he rolls, respectively. What is the expected value of $\frac{1}{br}$? Express your answer as a common fraction.

21. ________________ For real values x and y, let $x^2 - 4x + y^2 - 6y + 12 = 0$. What is the minimum value of $\frac{y}{x}$? Express your answer as common fraction in simplest radical form.

22. ________________ A wire mesh is formed from 27 units cubes that join together to form a 3x3x3 cube. A bug starts at the front left lower corner and travels exactly 9 units to reach the opposite corner. What is the probability that the ant will only walk on the surfaces of the cube (not go inside)? Express your answer as a common fraction.

23. ________________ How many ordered triples of integers (x, y, z) exist such that $x^3 + 5y^3 = 25z^3$?

24. ________________ What are the last 3 digits of $3 \cdot 7 \cdot 11 \cdot 15 \cdot \ldots \cdot 2015 \cdot 2019$?

25. __________________ Let ABC be a right triangle where $\overline{AB}$=10, $\overline{AC} = 6$, and $\overline{BC}$=8. Let circle O be tangent to AC and AB, and let circle P be tangent to CB, AB, and circle O. Circle O is congruent to circle P. Find the radius of circle O. Express your answer as a common fraction.

26.___________________ Tetrahedron ABCD has side lengths $\overline{AB} = \overline{BD} = 10, \overline{AC} = \overline{DC} = 17, \overline{BC} = 21$, and $\overline{AD} = 8$. Find the volume of tetrahedron ABCD. Express your answer in simplest radical form.

27. ___________________ The equation $2xy - 2x^2 - 5y + x + 4 = 0$ has exactly four ordered pairs (x, y) of integer solutions. What is the maximum value of $x + y$?

28.___________________ An *one-and-only number* is a number consisting only of the digit 1, such as 111 and 11111. Find the number of digits in the smallest one-and-only number that is divisible by 19.

29. ___________________ For all real numbers x, find the maximum value of $\sqrt{x^2 - 2x + 10} - \sqrt{x^2 - 8x + 17}$. Express your answer in simplest radical form.

30.___________________ In triangle ABC, $\overline{AD}$ is the angle bisector of acute angle A. $\overline{BE}$ is a median of $\overline{AC}$, and $\overline{BE} = 8$. $\overline{AD} = 12$. $\overline{AD}$ and $\overline{BE}$ are perpendicular. The perimeter of ABC can be expressed in $a + b \cdot \sqrt{c}$. Find the sum $a + b + c$.

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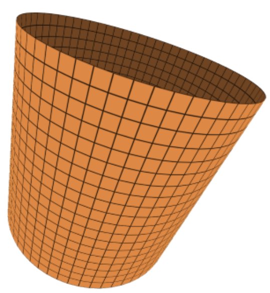
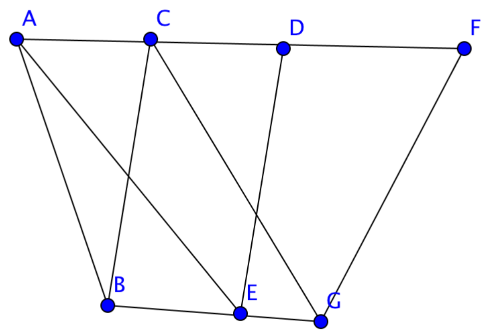
2017
Mock National Competition
Target Round
Problems 1-2

1.________________ 2016 evenly spaced vertical lines and 2017 evenly spaced horizontal lines are drawn on the lateral face of a truncated cone. If the number of quadrilaterals formed on the lateral faces is n, what is the sum of the digits of n? (The ‘quadrilaterals’ may be curved; the answer is not 0).



2.________________ How many quadrilaterals are in the following figure? (The vertices of the quadrilaterals have to be on intersection points, but don’t have to be on A, B, C, D, E, F, or G)



MATHCOUNTS®

2017

■ Mock National Competition ■

Target Round

Problems 3-4

3.________________ Let S be the set only containing the positive integers less than 10. How many ways can a subset P of S be chosen such that no two elements have an absolute difference less than 3?

4.________________ Richard is planning a trip to Orlando in May. He knows that his vacation will last six days. He checks the weather, and realizes there is a 75% chance of rain on the first three days and a 25% chance of rain on the last three days. If the MathCounts competition he is attending is on the last day of vacation, and will only be held if it rains at least two times in the previous days in his vacation, what is the what is the probability that the MathCounts competition will be held? Express your answer as a common fraction.

2017
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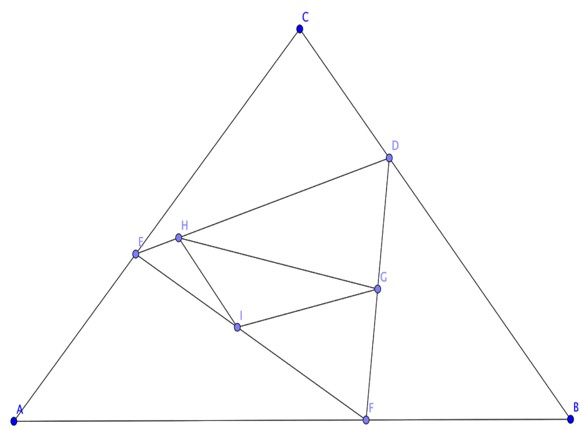
Target Round
Problems 5-6

5.________________ In triangle ABC points D, E, and F are chosen on sides $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, such that $\overline{AF} : \overline{FB} = 3:2$, $\overline{BD} : \overline{DC} = 2$, and $\overline{CE} : \overline{EA} = 3:2$ Additionally, points G, H, and I are chosen on sides $\overline{FD}$, $\overline{DE}$, and $\overline{EF}$, such that $\overline{EI} : \overline{IF} = 2:3$, $\overline{FG} : \overline{GD} = 1$, and $\overline{DH} : \overline{HE} = 5$. What is the ratio of the area of triangle DEF to the area of triangle ABC? Express your answer as a common fraction.



6. ________________ Given that real numbers a,b,c are all not zero, and $a+b+c=0$. Find the value of $x^2 - 2x + 2017$, where

$$x = -\left|\frac{|a|}{b+c} + \frac{|b|}{a+c} + \frac{|c|}{a+b}\right|.$$

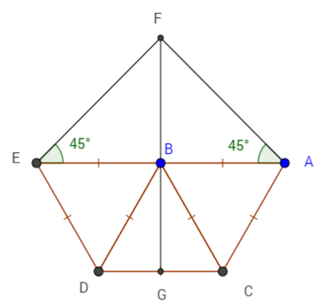
2017

Mock National Competition

Target Round

Problems 7-8

7.__________________ In pentagon ACDEF, DE = BE = BD = BC = CD = AB = AC, and m$\angle$FEB = m$\angle$FAB = 45 degrees. Pentagon ACDEF is rotated 360 degrees around line FG. If DE = 2, what is the volume of the resulting solid? Express your answer as a common fraction in terms of π in simplest radical form.



8._________________ A 1-inch thick cylindrical tube has an outer diameter of 6 inches, and has a height of 12 inches. 8 inches up from the base, the tube is sliced at a 30° incline angle, as shown below. What is the total surface area of the ring formed by the slice? Express your answer in terms of π.